Lesson 6. Cylinders and Quadric Surfaces

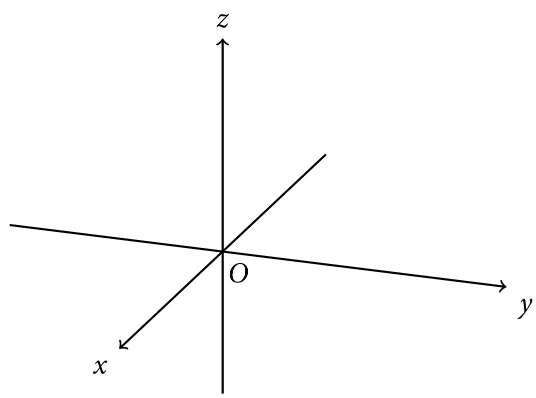
1 In this lesson...

- Special families of surfaces in 3D space
- Drawing different types of surfaces in 3D space

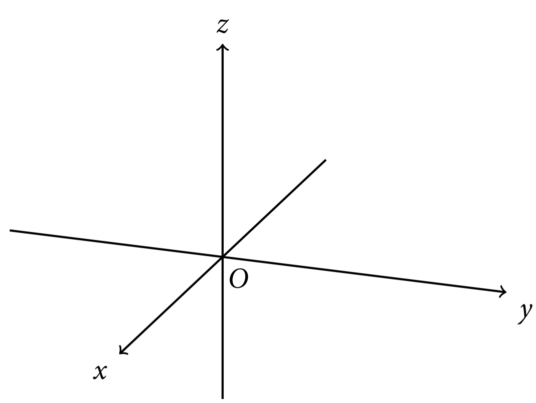
2 Cylinders

- A **cylinder** is a surface composed of all lines that
  - are parallel to a given line and
  - pass through a given plane curve
- In 3D, if one of the variables x, y, z is missing from the equation of a surface, then the surface is a cylinder

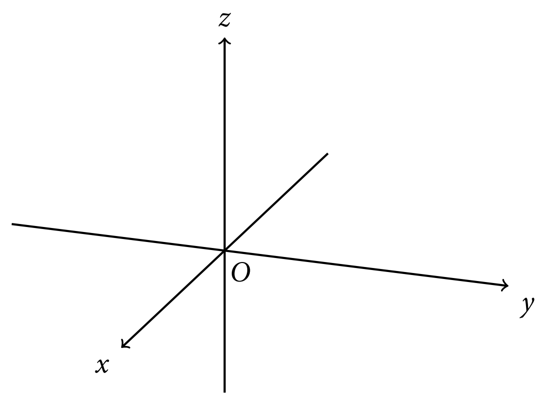
Example 1. Sketch the graph of the surface $z = x^2$.



Example 2. Sketch the graph of the surface $y^2 + z^2 = 1$.



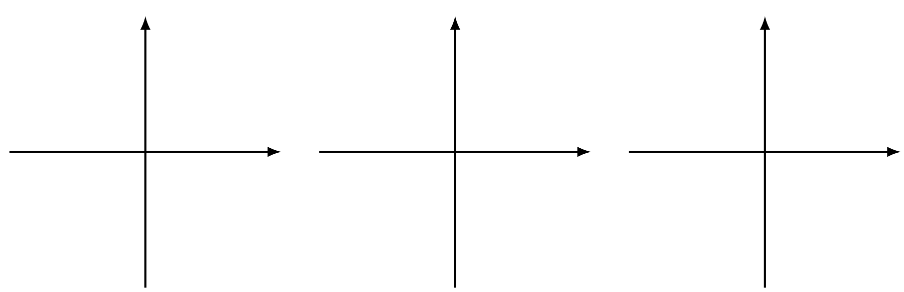
Example 3. Sketch the graph of the surface $xy = 1$.

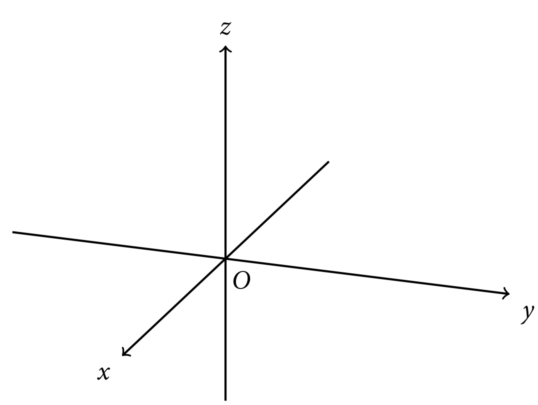


3 Traces

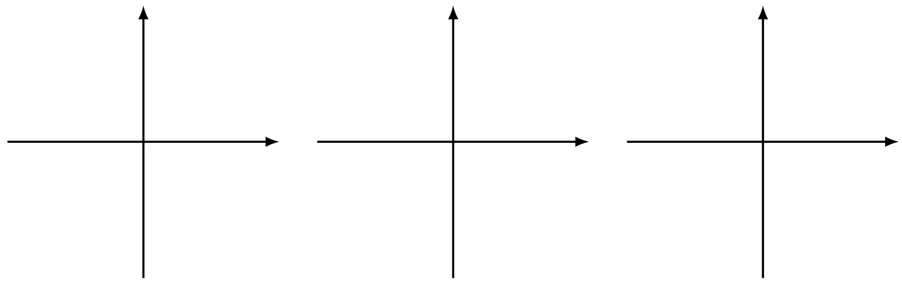
- A **trace** of a surface is the curve of intersection of the surface with planes parallel to the coordinate planes
- Idea:
  - Start with an equation in 3 variables x, y, z
  - Plug in a value for one of the variables
  - Graph the resulting equation in 2 variables (i.e., graph a trace of the surface)
  - Repeat for other values and other variables
  - "Glue" the traces together

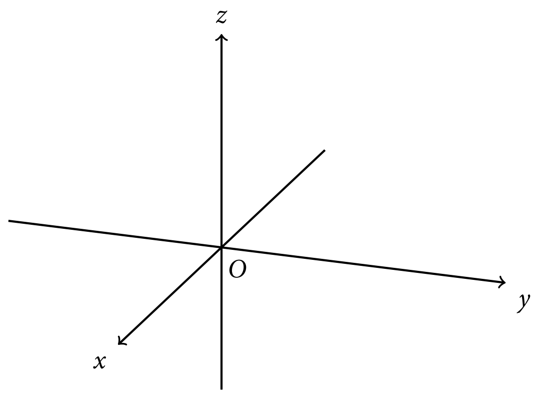
Example 4. Use traces to sketch the surface $z = 4x^2 + y^2$.





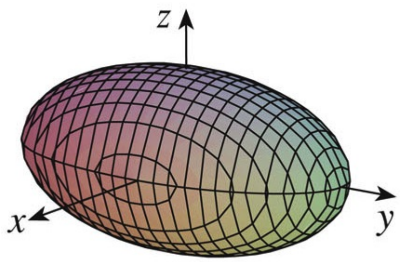
Example 5. Use traces to sketch the equation $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$.





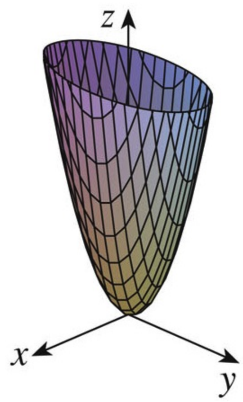
4 Quadric surfaces

- **Ellipsoid**



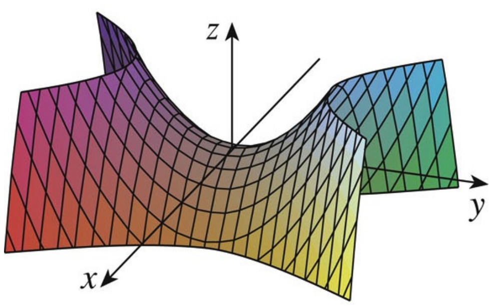
  - Equation:
  - All traces are ellipses
  - If $a = b = c$, the ellipsoid is a sphere

- **Elliptic paraboloid**



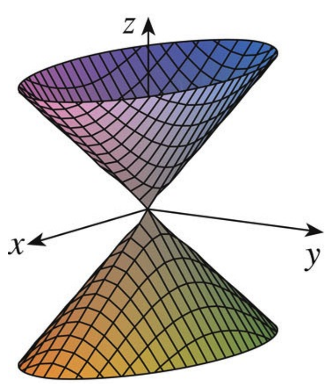
  - Equation:
  - Horizontal traces are ellipses
  - Vertical traces are parabolas
  - The variable raised to the first power indicates the axis of the paraboloid

- **Hyperbolic paraboloid**



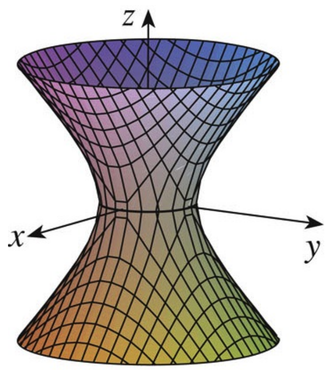
  - Equation:
  - Horizontal traces are hyperbolas
  - Vertical traces are parabolas
  - The case when $c < 0$ is illustrated

- **Cone**



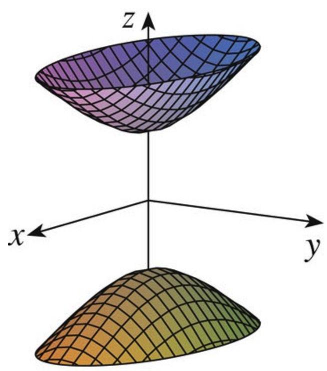
  - Equation:
  - Horizontal traces are ellipses
  - Vertical traces are planes or hyperbolas

- **Hyperboloid of one sheet**



  - Equation:
  - Horizontal traces are ellipses
  - Vertical traces are hyperbolas

- **Hyperboloid of two sheets**

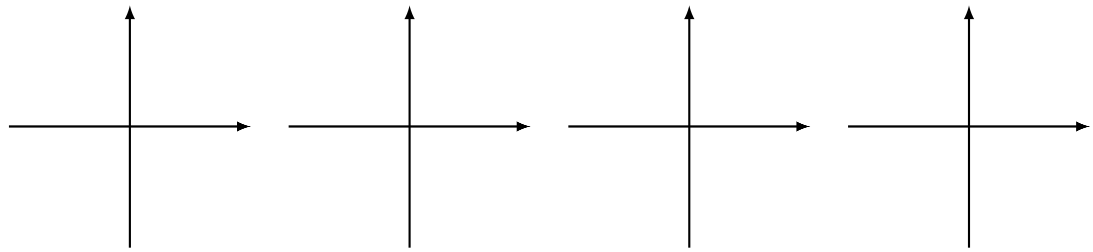


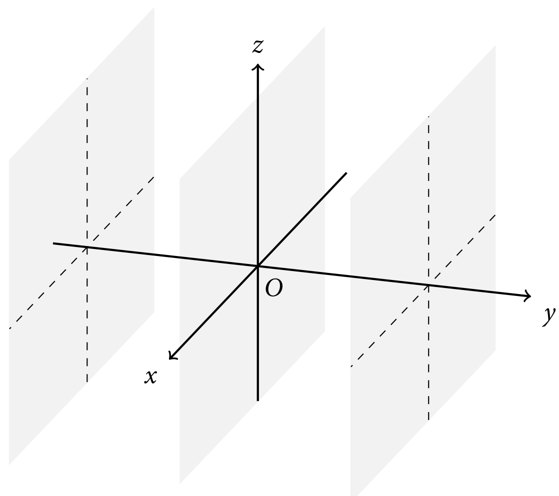
  - Equation:
  - Horizontal traces (when $z = k$) are ellipses if $k > c$ or $k < -c$
  - Vertical traces are hyperbolas

- Equations given above are in "standard form"
  - May need to do some algebra to get an equation into standard form
- Equations given above are for surfaces that are symmetric about the z-axis
  - May need to switch the variables around to match an equation with the surface type

Example 6. Sketch the quadric surface $z = y^2 - x^2$. What is this quadric surface called?

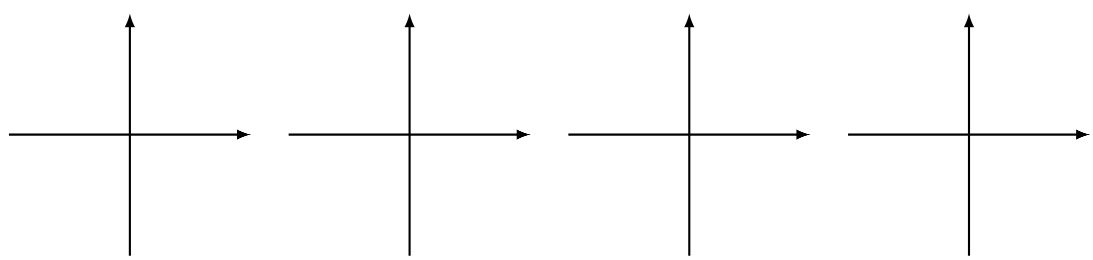
Hint. Draw traces for this surface when $x = 0$, $y = 0$, $y = 1$, and $y = -1$.

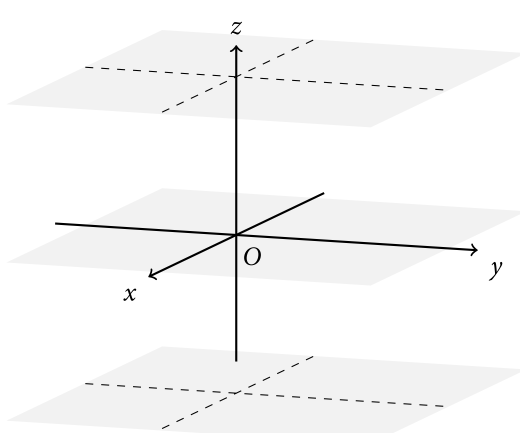




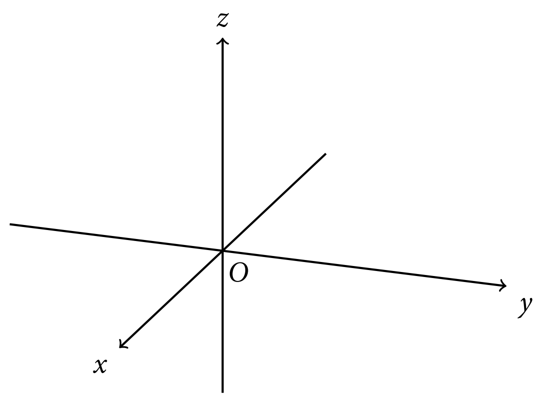
Example 7. Sketch the quadric surface $x^2 + y^2 - z^2 = 1$. What is this quadric surface called?

Hint. Draw traces for this surface when $z = 0$, $z = 1$, $z = -1$, and $x = 0$.





Example 8. Identify and sketch the quadric surface $2z^2 - 4x^2 - y^2 - 4 = 0$ by matching the equation to the standard equations given above.



Example 9. Identify and sketch the quadric surface $2y^2 = x^2 + 4z^2$ by matching the equation to the standard equations given above.